

Developing mathematical thinking in the primary classroom: liberating students and teachers as learners of mathematics

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Developing mathematical thinking in the primary classroom: liberating students and teachers as learners of mathematics

BRIAN HUDSON, SHEILA HENDERSON and
ALISON HUDSON

This paper reports on a research study conducted with a group of practising primary school teachers ($n = 24$) in North East Scotland during 2011–2012. The teachers were all participants in a newly developed Masters course that had been designed with the aim of promoting the development of mathematical thinking in the primary classroom as part of project supported by the Scottish Government. The paper presents the background for this initiative within the context of the Scottish Curriculum for Excellence reform. Particular attention is given to the epistemological positioning of the researchers as this influenced both the curriculum design process and also the theoretical framing of the research study which are both described. The project was set up within a design research framework, which aimed to promote classroom-based action research on the part of participants through the course and also research by the university researchers into the process of curriculum development. The research questions focused on the teachers' confidence, competence, attitudes and beliefs in relation to mathematics and their expectations and experiences of the impact on pupil learning arising from this course. Empirical data were drawn from pre- and post-course surveys, interviews and observations of the discussion forums in the online environment. Findings from this study highlight the way the course

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had a transformational and emancipatory impact on these teachers. They also highlight ways in which the ‘framing’ of particular aspects of the curriculum had an oppressive impact on learners in the ways that suppressed creativity and limited the exercise of learner autonomy. Furthermore, they highlight the ways in which a number of these teachers had experienced mathematics as a school subject in very negative ways, involving high levels of ‘symbolic violence’ and of being ‘labelled’.

Keywords: mathematical thinking; primary education; curriculum reform; epistemic quality; symbolic violence; labelling

1. Introduction

This research study was conducted with a group of 24 practising primary teachers from the local education authorities (LEA) of Angus, Dundee, Fife and Perth and Kinross in North East Scotland during 2011–2012. The teachers were all participants in a newly developed Masters’ level course that had been designed with the aim of promoting the development of mathematical thinking in the primary classroom. The curriculum development was carried out as part of project that was supported by funding from the Scottish Government during 2010–2012.

The background context for this initiative was the context of the Scottish *Curriculum for Excellence* reform that, in turn, has been located within the wider context of an international review of the quality and equity of education outcomes conducted by the Organisation for Economic Cooperation and Development (OECD, 2007). This report highlighted the fact that Scotland consistently performs at a very high standard in OECD’s Programme for International Student Assessment and has one of the most equitable school systems in the OECD. It also highlighted two major challenges, the first of these being an achievement gap that opens up late in primary education and widens through junior secondary years with children from poorer backgrounds being more likely than others to under-achieve. A second challenge highlighted the need to achieve broader and more successful participation in upper secondary education and greater equity in higher education. The curriculum in Scotland is non-statutory and the Government’s responsibility is to provide the framework for learning and teaching rather than to prescribe or try to micromanage what happens in individual schools. The responsibility for what is taught rests firmly with local authorities and schools working together whilst taking account of national guidelines and advice. Whilst this curriculum is much less prescriptive than previously was the case and gives teachers more freedom to decide what to teach and how, it also places greater responsibility on teachers to lead its development.

The project proposal was submitted against a background in which, despite past initiatives to improve the teaching and learning of mathematics, most mathematics lessons in Scotland still tend to feature some form of teacher-led demonstration followed by children practising skills and procedures from a commercially produced scheme (Scottish Executive Education Department, 2005). These findings were confirmed by TIMSS (IEA, 2008) which found that 72% of both Primary 5 (P5) and

Secondary 2 (S2) pupils were taught using a textbook as the primary resource compared to the international average of 65 and 60%, respectively. The most recent Scottish Survey of Achievement (Scottish Government, 2009) at the time also reported that pupils using textbooks and working quietly on their own was the most common form of activity in mathematics classes in Scotland. More recently, the Scottish Survey of Literacy and Numeracy reported the activities in which the highest percentage of pupils reported that the ways in which they participated ‘very often’ were to ‘listen to the teacher talk to the class about a topic’ (62% in P4 and 64% in P7 and S2) and to ‘work on your own’ (between 55 and 61%) (Scottish Government, 2012, p. 13).

This background context suggests that attempts to move to more constructivist models of teaching and more active approaches to learning mathematics have not been as successful as might have been hoped. In turn, this informed the rationale for the project which aimed both to reduce the achievement gap and also to support the development of more engaging approaches to the teaching and learning of mathematics for all. Also of influence on our thinking at the outset was the Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools in England by Sir Peter Williams in 2008. This report called for improved subject knowledge of primary teachers in combination with pedagogical skills in order to promote effective learning of mathematics. Earlier work with student teachers had highlighted the importance of subject knowledge and teacher confidence (Henderson & Rodrigues, 2008) and also had raised questions on the nature of mathematics and highlighted the importance both of teachers’ beliefs and the affective dimension for student learning (Henderson & Hudson, 2011). These findings were consistent with other published work in the field, such as that by Bibby (2002) who explored similar questions using the central construct of ‘shame’ as a way of taking into account the ‘complete mosaic of mathematical beliefs’. The recommendation made in the Williams report was that ‘there should be at least one Mathematics Specialist in each primary school, in post within 10 years, with deep mathematical subject and pedagogical knowledge, making appropriate arrangements for small and rural schools’. This project followed a similar direction to that of the Mathematics Specialist Teacher (MaST) Programme in England that arose as a consequence. Related research evaluating the MaST programme is only now becoming available, for example, the case study reported on by Milik and Boylan (2013).

2. Epistemological positioning: the nature of mathematics

In considering the history of the development of the discipline of mathematics, it can be seen that the nature of the subject itself has long been contested. This has profound implications for school mathematics—for example, is it an abstract subject for an elite or should mathematics be for all? In the analysis contained in his seminal text Lakatos (1976, p 5) distinguishes between the deductivist approach and the heuristic approach,

which he describes as ‘the logic of proofs and refutations’. He presents the perspective of *mathematical fallibilism* based on a view of mathematics as human activity and on the proposition that it is this human mathematical activity that produces mathematics. In contrast, he argues that Euclidean methodology developed a certain obligatory style of presentation, which he describes as a deductivist style. Mathematics is compared with a conjuring act in which the student is obliged to accept this without asking questions about the underlying assumptions. In this deductivist style, under which all propositions are true and all inferences valid, mathematics is presented as an ever-increasing set of eternal, immutable truths.

However, we argue that it is not simply the dominating influence of this deductivist approach which is a main problem for the teaching and learning of mathematics in schools today but rather the way in which this can become distorted in the process of ‘didactic transposition’ (Schneuwly, 2011). The concept of ‘didactic transposition’ relates to the school context, in which the knowledge in question is not knowledge for acting and solving problems in the social contexts in which it was created and where it is used, but it is instead transposed into knowledge to be taught and to be learned. The concept of didactic transposition is based upon recognition that there is a ‘rupture’ between daily life and school, which changes the knowledge profoundly.

It is further argued that this rupture can lead to the *epistemic quality* of the subject becoming degraded as it is transposed into school mathematics. Aspects that are associated with *high epistemic quality* involve an approach which presents mathematics as fallible, refutable and uncertain and which promotes critical thinking, creative reasoning, the generation of multiple solutions and of learning from errors and mistakes. In contrast, *low epistemic quality* is characterized by an approach that presents the subject as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain and which involves rule following strict procedures and right or wrong answers. Furthermore, high epistemic quality is promoted through an approach based on assessment for learning involving low-stakes formative and self-assessment which is engaging and motivating for individual learners and which leads to a sense of enjoyment and fulfilment of mathematics as a creative human activity. In contrast, the excessive pressure from high-stakes external testing and inspection and the associated heavy emphasis on drill and practice lead to the degradation of epistemic quality into a form of *mathematical fundamentalism* and to an experience for learners of mathematics that is fearful and anxiety inducing, boring and demotivating and which leads to alienation from the subject itself.

When combined with a heavy emphasis on drill and practice, an over dependence on the use of textbooks adds further to creating the conditions for low epistemic quality. As Lakatos (1976, p. 146) argues, when mathematics is presented in textbooks this product of human activity ‘alienates itself’ from the very human activity that produced it. Similarly, in considering the place of geometry, Giles (1982, p. 37) referred in a very vivid way to the way in which ‘dead geometry’ becomes ‘entombed in text books’. This stands in stark contrast, for example, with the present-day opportunities afforded by the use of dynamic geometry software

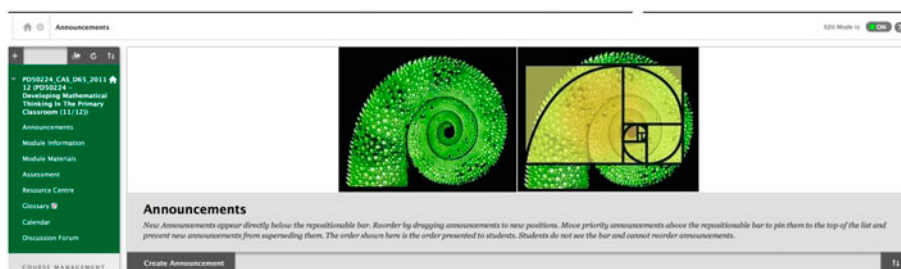


Figure 1. Welcome page for the online learning environment.

to enable students to independently study the invariant (unchanging) relationships between points, lines and circles, forming their own conjectures and testing them out visually, which is the very essence of geometry and spatial reasoning.

3. Curriculum development project

The overall aim of the curriculum development project was to develop and implement a Postgraduate course of Continuing Professional Learning on the theme of ‘Developing Mathematical Thinking in the Primary Classroom’ (DMTPC, 2011). The course was designed in the first year of the project (2010–2011) to form the first stage in a Masters level programme in Mathematics Education for Primary Specialists in Mathematics and to be accredited as a free-standing module worth 30 credits leading towards a Postgraduate Certificate in Developing Mathematical Thinking. The Curriculum Development Partnership Group that planned the course included one teacher with one from each of the LEA and two Local Education Authority staff members with one from Fife and the other from Angus. The course design was based on a blended learning model with an online learning environment established in the University Virtual Learning Environment (figure 1).

The module was opened in the online environment in September 2011 and was followed closely by the first-day workshop. There were two twilight sessions as part of the course that took place in October and December and a final-day workshop, which took place in February 2012. For those completing the associated assignment, the submission date was set for April 2012, which marked the end date of the course of study. The group was made up of 23 females and one male, with an age range from 21 to 55.

The course of study was structured around three key questions, two core texts and an action research project. The key questions were:

- (1) What is mathematics?
- (2) What is mathematical thinking?
- (3) What is good mathematics teaching?

The core texts were those by Boaler (2009) and Mason, Burton, and Stacey (2010) and the course assignment was the action research project

report. The latter was submitted for the award of 30 Masters level credits in the form of a report of 5000 words. Participants were required to design a project that would consider the implications of current literature on the development of mathematical thinking; identify strategies, skills and attitudes to be developed; identify methods of data collection and analysis that provide evidence of impact; be feasible and realistic in terms of the resources available in school; give consideration to progression and assessment and identify the issues which emerged from the study.

From the outset of the course, it became clear that the reading by the participants of *The Elephant in the Classroom* (Boaler, 2009) had invoked very powerful responses amongst this group of teachers. In the introduction to the book, Boaler writes:

I have called this book ‘The elephant in the classroom’ because there is often a very large elephant standing in the corner of maths classrooms. The elephant, or the common idea that is extremely harmful to children, is the belief that success in mathematics is a sign of general intelligence and that some people can do maths and some can’t. Even maths teachers (the not so good ones) often think that their job is to sort out those who can do maths from those who can’t. This idea is completely wrong and this is why. In many maths classrooms a very narrow subject is taught to children, that is nothing like the maths of the world or the maths that mathematicians use. This narrow subject involves copying methods that teachers demonstrate and reproducing them accurately over and over again. Of course very few people are good at working in such a narrow way, and usually everyone knows which people are good at it and which people are not. But this narrow subject is not mathematics, it is a strange mutated version of the subject that is taught in schools. When the real mathematics is taught instead—the whole subject that involves problem solving, creating ideas and representations, exploring puzzles, discussing methods and many different ways of working, then many more people are successful. (Boaler, 2009, p. 2)

This phenomenon reflects that highlighted in the seminal work of Richard Skemp (1976) who wrote from a psychological perspective about there being two effectively different subjects taught under the name of mathematics. His work highlighted the way in which such contrasting teaching approaches (based on teaching ‘real’ or ‘narrow’ mathematics) resulted in ‘relational’ or ‘instrumental’ understanding.

A keynote session in the first workshop was structured around the second text entitled *Thinking Mathematically* (Mason et al., 2010). The authors outline an approach to thinking mathematically which rests on five important assumptions which are that anyone can think mathematically, that mathematical thinking can be improved by practice with reflection, that it is provoked by contradiction, tension and surprise, that it is supported by an atmosphere of questioning, challenging and reflecting and that it helps in understanding oneself and the world. Improving mathematical thinking is seen to involve tackling questions conscientiously, reflecting on this experience, linking feelings with action, studying the process of resolving problems and noticing how what one learns fits in with one’s own experience. The influence of the mathematician George

Pólya is apparent and he is acknowledged by the authors as one of their major sources of inspiration.

Physical, intellectual and emotional involvements are all seen as being necessary to enable mathematical thinking. A central idea relates to that of 'being stuck' which is often the barrier at which progress can stall. However, this is an almost inevitable stage for most learners and is an essential state to reach for the development of improved mathematical thinking. The key to breaking through this barrier is seen to be through learning to cope with the emotions aroused through 'being stuck' which involves taking time: time to ponder the question; time to try several different approaches and follow different paths, though some approaches and paths will present blind alleys and many will not provide immediate insight.

Instead of feeling a sense of disappointment at a particularly difficult problem, attempts should be made to learn from unsuccessful attempts. This approach involves the use of a range of techniques and reflection on experience. Processes are stressed rather than outcomes or 'solutions'. The term 'resolution' rather than 'solution' is the preferred description of a successful outcome. It is stressed that the elegant solution that is so often found in mathematics textbooks rarely arises spontaneously but is the product of many blind alleys, tortuous periods of barren thinking and unexpectedly sudden insights. Furthermore, such outcomes are then subject to many iterations and refinements before arriving in their final polished and elegant form in the final textbook.

With regard to the underlying processes of inquiry, mathematical thinking is seen to be founded on the processes of *specializing* and *generalizing* that lead to those of *conjecturing* and *convincing* in a broad overlapping spiral of cycles as outlined in figure 2.

In order to support the processes of inquiry, techniques are used for recording mathematical experience. These are intended to enable the learner to analyse and study this experience later and also to notice (become aware of) key stages and phases in their own mathematical thinking. These techniques are based upon writing things down as notes to oneself with the aim of recording all the significant ideas that occur in the search

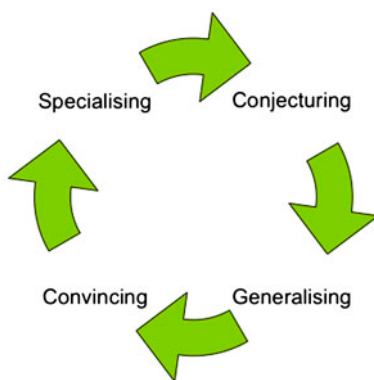


Figure 2. Processes of inquiry underlying mathematical thinking.

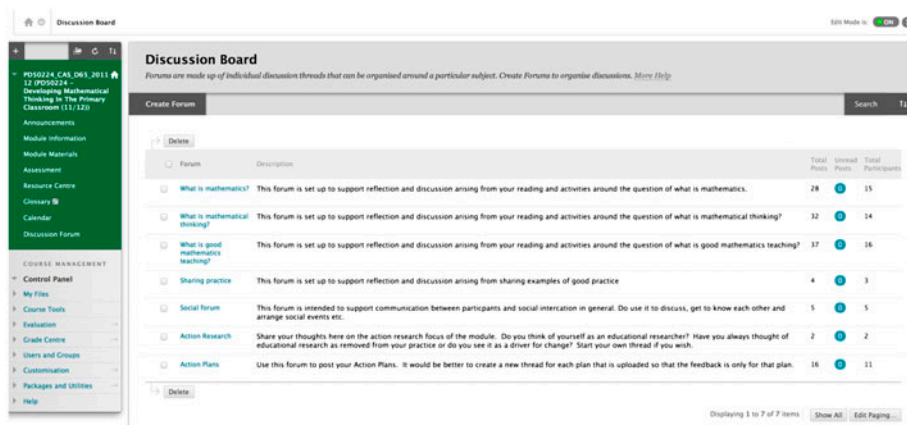


Figure 3. Discussion board in the online learning environment.

for the resolution to a question, what exactly one is trying to do and also one's associated feelings. This is seen as a challenging task but with time and practice as one through which these techniques become increasingly internalized.

Discussions around the three key questions were structured into the study process and supported by discussion forums in the online learning environment as shown in figure 3.

There were 28 contributions from 15 participants to the discussion forum on *What is mathematics?* 32 contributions from 14 participants to the discussion forum on *What is mathematical thinking?* and 37 contributions from 16 participants to the discussion forum on *What is good mathematics teaching?* Discussions that addressed the first question about the nature of mathematics took place over the first month of the course until the end of October whilst discussions around the second two questions continued until the beginning of February. Limitations of space do not allow for a full consideration of the contents of the discussion board in this paper. However, the following comments provide a flavour of some of the discussions, which are taken from those around the nature of mathematics. All of the contributions offered reflections on early experiences as learners of mathematics and of the need to continually rebuild confidence and re-learn mathematics in later life as a teacher.

I feel empowered having read both articles. I believe that maths should be taught within context giving relevance to the children's learning whilst allowing them to discuss and share ideas and solutions to problems. It is the creativity element that I feel I struggle with. (Anna)

I agree that maths should be exciting, inspiring and contextualised. This was not my personal experience of this subject and if truth be told, I am left with the scars of old fashioned methodology. (Joan)

When I think about What is Mathematics?—I go back to my own school experience and can totally relate to the presentation at Saturday's workshop

and how much emotion is tied up with experiences of Maths. Although I feel I experienced success at primary school- my high school experience was less enjoyable. After reading Boaler's book I can completely relate to being one of those girls that asked—why? and tried very hard to relate what I was doing in maths class to my every day life—and I just couldn't make the connections—so I gave up going any further with maths as soon as i realised you didn't need a higher qualification for teaching! As a result any 'challenging' maths and i feel myself going into panic mode! Fight or flight! And very easily giving up! In contrast to this I really enjoy teaching maths—although most of my experience is infant maths! However more recently as Support for Learning teacher I have been stretched in my teaching skills with able children! And all those feelings of panic return! ... And I think as soon as you feel out of your comfort zone as a teacher—you tend to rely on textbooks! So for me Mathematics is a subject that can be fun to teach but also one to be feared! (Jane)

Maths is a subject I have always enjoyed although not always understood ... I'll admit to being a bit of a geek in that once I started reading Boaler in the summer holidays the 'quick skim' turned into my regular night time reading—I couldn't put it down! It made so much sense and it genuinely inspired me to put into practice some of the ideas I'd had for my class but didn't quite have the confidence to follow through. (Chloe)

I absolutely agree. As I read Boaler's book, I kept thinking back to my High school experience. I felt frustrated, stupid and kept wanting to know WHY did I have to learn what 'x' is, what did it mean, what was the reason? So, I also related to the girls in the study, I was one of them! I think we teach too much from text books but making the jump is difficult. I want to get it right, make maths enjoyable and fun. But, what I want most is to not let the past repeat itself with another generation leaving school frustrated and alienated from maths. (Gemma 1)

It is hard to unpick what I thought mathematics was before I started reading for the course and attending the workshop and what I am beginning to formulate and develop in light of thinking about Maths in a new light. I have pattern, problem-solving tools, something that is all around us mixed up with numeracy teaching in schools. I do know that it is a subject that children appear very excited about when they are young and that this can change to fear and loathing as they progress through school. It is a subject that is perceived as one which you are either innately good at or have no flair for. Often described as hard. Having right answers. Boring. Irrelevant.

For me it is a subject that I have grown to like more and more and understand better the longer I have been teaching. Like (Jane), at school, I was a pupil who wanted to know 'Why?' and when I received no answers became disengaged in the part that I would have probably found really interesting. (If anyone can tell me why i might need to know the formula for describing a doughnut round the x axis I'd love to hear from them!) I do remember loving solving equations and geometry. Later, as a History of Art student I was fascinated by the golden ratio in the development of architecture. I still marvel at patterns such as the Fibonacci sequence and the question I always want to know is who discovered that? How did they go about it? What does that sort of maths look like? Mathematics is a subject that I

would love to teach in a way that keeps children fascinated and help them build resilience as learners. (Jenny)

What we want it to become is an enjoyable and meaningful mainstay of teaching, and reading these books is certainly inspiring me to think that this could be possible for the children. (Pauline)

I have just re read the articles and feel that I need to take some risks ... I think as teachers our pupils see us as the experts and I wonder how they would view us if we shared with them that we 'can also struggle ... that this is normal.' The important point is that we need to be thinking out loud and acknowledging we all get stuck some times. The 'creative' side of maths has been mentioned in both articles and I have never thought of maths in that way. That is my personal challenge and the one I struggle with. (Gemma 2)

As a teacher I used to think maths had a right or wrong answer but reading Boaler made me re think my views and practice. I feel my head is trying to grapple with different thoughts and opinions as I have a dichotomy view of maths. My early (school and twenty year old) view, when I hated maths, and my older working view point through real life usage and becoming a teacher. I am trying to filter out my own past anxieties of maths while enjoying the teaching of maths. She reminded of my own conceptual difficulties with maths but she also offered a way forward. That really excited me. (Gemma 3)

With regard to the action research project, all participants took part in the first planning stage of this process through undertaking an audit of their practice, supported by input from the LEA support staff. This was then used as the basis of the development of an action research plan for a small-scale project designed to develop mathematical thinking in their classrooms. Accordingly, formative tutor feedback was given on action plans, which were submitted via the online learning environment. The second stage involved undertaking the project and completing the writing up of the Action Research Project report. In their reports, participants were required to reflect upon the role of the teacher in developing mathematical thinking and promoting good mathematics teaching; indicate how the study had impacted on classroom practice and children's learning with reference to the supportive evidence and to conclude with an evaluative summary of the their own learning from the study and how this might be developed further in the future.

All participants put their ideas into action by carrying out a classroom-based inquiry of which 10 submitted Action Research Project reports for formal assessment. Members of the tutor team assessed the reports in May 2012, and all were passed successfully subject only to the need for some minor technical corrections. The projects addressed a diverse and highly relevant range of topics and questions as outlined below:

- (1) Meeting the challenges of mixed ability mathematics in a multi-composite class.
- (2) An inquiry into the use of non-commercial resources to develop children's mathematical thinking.

- (3) Open-ended activities and encouraging collaborative talk with 7–8 year olds.
- (4) Using real-life mathematics in the everyday teaching of the subject to engage and develop mathematical thinking.
- (5) An examination of the effects of pupils carrying out open-ended investigations in mathematics and using a variety of media to communicate their mathematical solutions and ideas.
- (6) Using questioning to extend the mathematical thinking of infants and to increase the children's ability to decompose and recompose numbers.
- (7) To what extent does topic-based mathematics allow children to demonstrate their mathematical thinking?
- (8) How does investigating proofs with Primary 2 develop mathematical thinking?
- (9) A collaborative investigation into developing mathematical thinking—making connections.
- (10) How do the teacher's questioning skills have an impact on mathematical thinking?

The report of the study on topic-based mathematics (7) by Anna was judged to be an outstanding assignment. This was identified to be an exemplar of the role of the teacher as researcher at its best, involving innovative and creative planning, spontaneous and responsive classroom action, finely grained classroom observation and insightful evaluation. Formative and summative feedback to Anna was structured in response to the assessment criteria, as was the case for all participants. The general aspects of the assessment were based on six criteria involving the ability to reflect critically on professional practice, as evidenced by appropriate extracts from portfolios and research journals; commitment to construction and reconstruction of professional practice taking into consideration social and cultural contexts and underpinning philosophies and policies; the ability to discuss and debate ideas, policies, strategies and actions, informed by research in the subject area; up-to-date knowledge of literature in the subject area related to the professional context; the undertaking of a professionally significant inquiry with the findings following research reporting conventions integrated into their assessment paper and the quality of presentation, structure and conventions of academic writing including evidencing and referencing. Summative feedback from the tutor included the following:

You have written a really interesting, excellent and exemplary action research report. It is very well structured and includes a well-written abstract. Your writing style is clear and succinct and draws very effectively on your reading about mathematical thinking. In particular you integrate the phases of Entry, Attack, Review in a very effective way with your topic-based approach. This approach is really imaginative and is richly described as a result of very close and detailed classroom observations. I really like your classroom accounts and examples in relation to the question of comparing the River Tay with the Amazon River! Your use of images of pupils' work brings the report to life in a very vivid way.

The case study featured in this report is considered in detail in Hudson (2013).

4. Research design

The DMTPC project as a whole was established within a design research framework as indicated in outline of the project plan figure 4.

Such an approach aims to support the construction of propositions for actions in relation to teaching and learning and to design and construct teaching situations, pedagogical activities and learning environments that enable both teachers and learners to put these propositions into practice. In this case, the focus of development has been primarily on teachers' professional learning. The first phase of the design research framework took place in the first year of the project involving informed exploration and the design of the instruments and tools for gather data. This was carried out alongside the process of curriculum development as captured in the second row and also alongside the recruitment of the participants. Further phases took place in parallel implementation or enactment of the module and subsequently in parallel with the action research projects carried out by the participants.

4.1. Research questions

The key research questions addressed were:

- (1) What are the teachers' perceptions concerning their levels of confidence and competence in relation to teaching mathematics?
- (2) What are the teachers' attitudes and beliefs in relation to mathematics as a subject?
- (3) What are the teachers' expectations of the impact on pupil learning arising from this course of study?

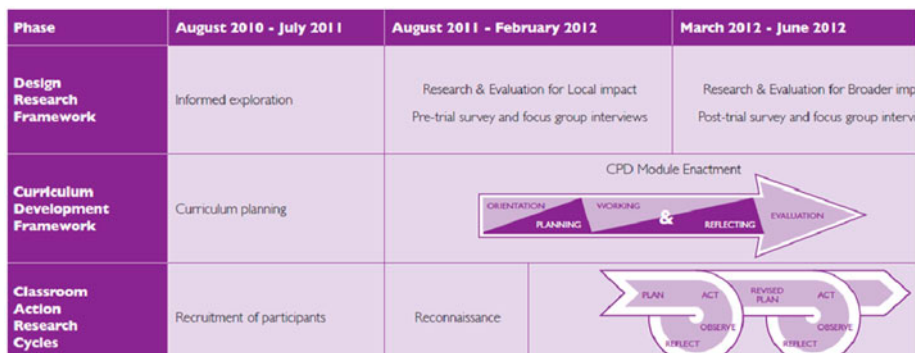


Figure 4. DMTPC Project Plan.

- (4) How do these perceptions, attitudes, beliefs and expectations change as a result of participating in this course of study?

4.2. Research methods

Quantitative and qualitative empirical data were drawn from pre- and post-course surveys of the teachers' perceptions, pre- and post-course interviews with a selection of participants, observations of engagement in the online environment, document analysis of audits of classroom practice and action plans for classroom inquiry. This paper draws on observations on one aspect of the online discussions related to the nature of mathematics, the pre- and post-course surveys and also on the pre- and post-course interviews with a selection of participants.

The pre- and post-course surveys were based on a series of statements drawn from a juxtaposition of the characteristics of the 'fundamentalist' and 'fallibilist' views of mathematics on a spectrum as discussed earlier. Accordingly, participants were asked to indicate *Strongly Disagree*, *Disagree*, *Agree* or *Strongly Agree* against a number of statements based on this model. In addition, they were asked their ages, how many years they had been teaching, the primary stage they usually teach and their highest level of mathematics qualification. Finally, in the pre-course survey the teachers were given the opportunity to comment on what had prompted them to embark on the course and in the post-course survey their experiences of the course.

The pre- and post-course interviews were conducted with four participants who volunteered and were drawn from each of the four participating LEAs. The interview questions focused on teachers' perceptions concerning their levels of confidence and competence in relation to teaching mathematics, their attitudes and beliefs in relation to mathematics as a subject and their expectations of the impact on pupil learning arising from this course of study.

4.3. Theoretical framework for analysis

A thematic analysis of the qualitative interview data was first carried out which was then followed by a reflexive discussion within the team. This process highlighted the need for critical analyses drawing on the theoretical lenses provided by a sociological perspective. In particular, we found the use of Bernstein's (2000) concept of 'framing' and Bourdieu's (1998) concept of 'symbolic violence' to be illuminating in the analysis of our data. These theoretical frameworks were found to be especially helpful in relation to the emergent themes of 'being allowed' and 'labelling' in particular. The concept of framing is used in relation to that of classification, which is used to describe the power relationships between different categories, for example, subjects, teachers, departments and practices. Depending on the degree of insulation between these categories, classification is considered as being strong or weak. Strong classification between

subjects implies little or no collaboration between teachers across subject boundaries whereas weak classification may lead to greater integration across subjects and more collaboration between teachers. The categories hold relative positions of power and any attempt to change the degree of insulation will reveal the power relationship on which the classification is based (Bernstein, 2000). The concept of framing in education is a way of describing the locus of control over the selection, sequencing, pacing and criteria for evaluating or assessing what counts as legitimate knowledge. Framing is about 'principles of control within the school context'. A major issue in this project has been the locus of control, between central regulation and the individual teacher, and degree of control over organization, selection, sequencing, pacing and criteria for the evaluation of knowledge to be acquired. So, for example, if the framing is strong there are national regulations that leave limited possibilities for teachers to plan their teaching practice, whereas weak framing provides teachers with the freedom to choose their methods of teaching and learning.

The work of Bourdieu (1990) offers an approach to understanding of practices in professional fields through the concepts of field, habitus, position, agency and capital. Fields are identified as social arenas in which capital is accumulated and where struggles for power, position and resources take place. Habitus is a set of deeply founded dispositions and beliefs rooted in the daily practices of individuals and groups, which contribute to the accumulation of capital and the exercise of agency. Social agents are seen as holders of specific capital who, through habitus, actively contribute to the construction of the field. Agency meanwhile refers to the capacity and position of individuals to act within a particular field. Bourdieu argues that social agents as subjects and, through habitus, acquire a sense of the game they engage in. Bourdieu offers two further concepts of particular interest to this study, which are those of symbolic capital and symbolic violence. Bourdieu argues that the struggles of agents to identify with one group or another or to be differentiated from one group or another, is a struggle over symbolic capital.

The concept of symbolic violence is used by Bourdieu to identify the existence of hidden struggles activated socially by a holder of capital as a means of limiting and controlling the agency of individuals with less capital. Symbolic violence is a 'soft' form of violence, which can be conveyed through looks, gestures and body language, or denied for reasons of self-interest. Social agents may favour acceptance of 'the world as it is' and/or find the dominant order 'natural' because 'their mind is constructed according to cognitive structures that are issued out of the very structures of the world' (Bourdieu and Wacquant, 1992, p. 168). Furthermore, Bourdieu argues that symbolic violence 'is the violence which is exercised upon a social agent with his or her complicity'. (Bourdieu, 1998, pp. 167–168) In other words, symbolic violence is about how the dominated come to accept their own conditions of domination (Swartz, 1997). The concept of 'symbolic violence' thus provides a critical lens through which to view the positioning of individuals and in particular the interplay between those who supposedly 'can do' mathematics and the stigmatized learners who supposedly 'can't'.

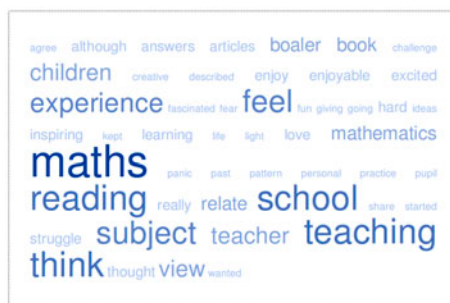


Figure 5. Word Cloud from discussion forum *What is Mathematics?*

4.4. Research ethics

Research ethics approval was gained from the University Research Ethics Committee on the basis of a summary outline of the project together with Participant Information Sheet and Consent form. Since the group was made up of 23 females and 1 male, female pseudonyms are used throughout in order to ensure that the anonymity of the single male participant is not compromised.

5. Results

5.1. Observations of the discussions in the online environment

Of the seven highlighted contributions to the discussion forum *What is Mathematics?* four make reference to very negative early experiences as learners of mathematics, especially at secondary school level. This phenomenon was reflected also the through responses through the surveys and interviews. The Word Cloud shown in figure 5 shows the most common words referred in this discussion forum.

It is notable that one of the most common references is to how participants ‘feel’ about the subject with references to both positive and negative emotions. For example, participants referred to challenge, creativity, enjoyment, excitement, inspiration, love, fascination and fun in relation to the positive aspects. In contrast, aspects of fear, panic and of the subject being hard were highlighted. Reference is also made to the struggle that can be involved in the subject though this is also recognized as a ‘normal’ part of doing mathematics.

5.2. Pre- and post-course surveys

All participants completed the pre-course survey and the post-course survey was completed by 63% of them.

In the free response to the pre-trial survey, one participant wrote:

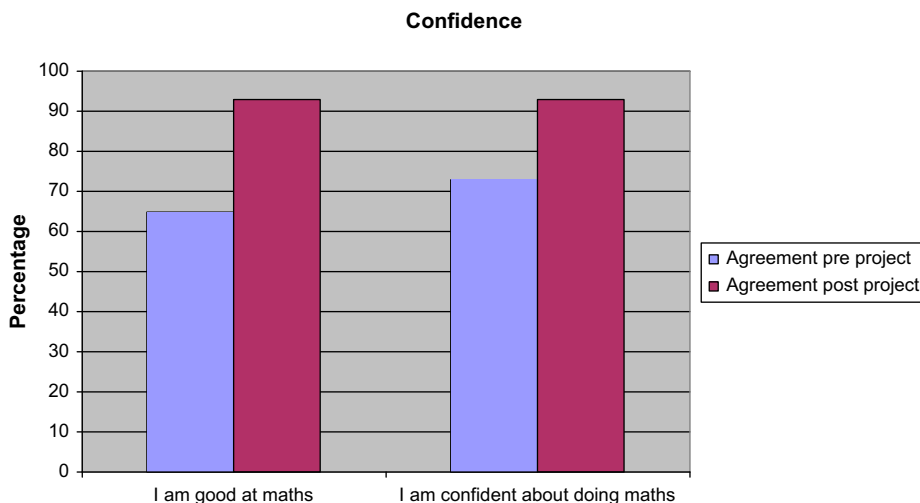


Figure 6. Pre- and post-course levels of perceived competence and confidence.

I wish I could understand maths because I find it so interesting but I have never really managed to be good at it. I am fascinated by the relationship between maths and the universe and practical disciplines such as engineering and science. The more I learn about maths the more it interests me but I feel somewhat frustrated that I cannot articulate this excitement to others. I frequently inhale the second hand mathematical knowledge of others but fail to understand even a fraction of what they clearly understand. I hope that this module will teach me more about what is essentially language.

Another participant wrote:

Many children fear maths and I would like to make it a more motivating subject where they enjoy what is being taught and what they are learning.

A third participant wrote:

Heard about it at a Sensation CPD, which I had enjoyed and thought it sounded promising. In school we are trying to make the move away from a textbook driven approach and I felt this module linked in well. I would like to broaden my teaching to much more activity based and co-operative learning styles in maths.

A fourth participant wrote:

I want to improve my practice and have children being active and excited about maths.

Through the structured section, participants were asked about their levels of perceived competence and confidence in mathematics pre- and post-course and these results are displayed in figure 6.

While 65 and 73% of participants already reported that they were good at and confident about mathematics before the start of the course, these figures rose to 93% in both categories by the end of the course.

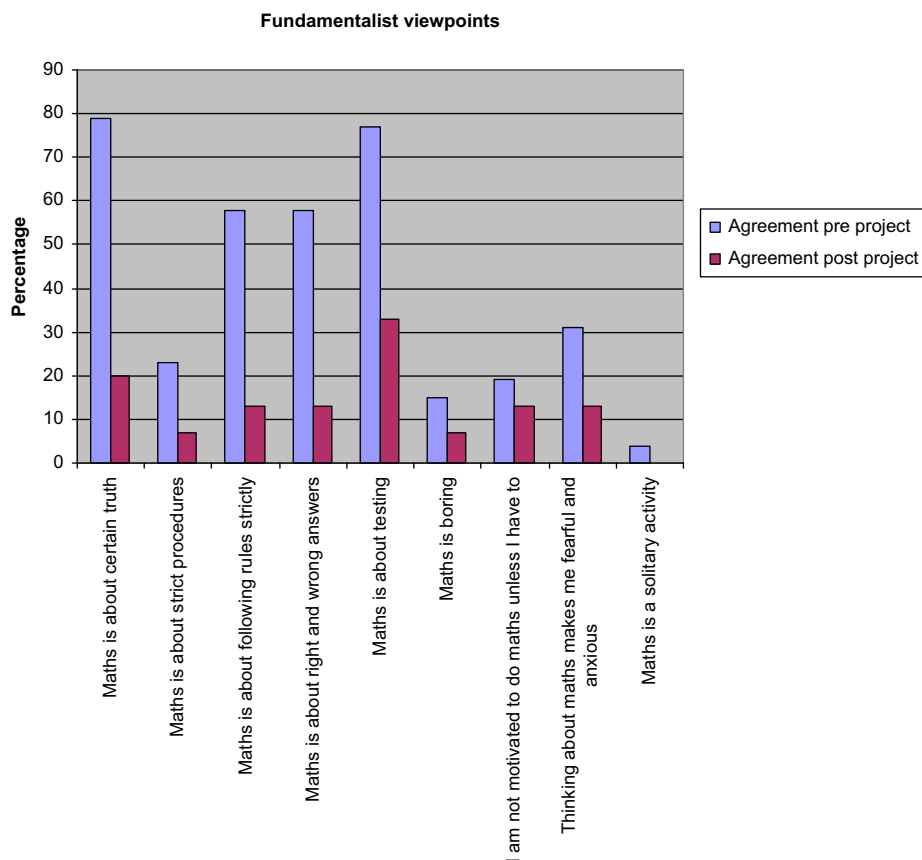


Figure 7. Changes in fundamentalist viewpoints pre- and post-course.

With regard to attitudes and beliefs, as can be seen in figure 7, there was a distinctive move from the fundamentalist viewpoints held by many of the participants at the start of the course, in particular those relating to the absolutist nature of the subject regarding rules, right and wrong answers and testing. With the move to more collaborative work in primary schools, it is not surprising to see that there was little agreement before the course and none after for the statement 'Maths is a solitary activity'. It is also interesting to note that despite many of the fundamentalist viewpoints held, only 15% of participants reported that they found mathematics boring before the course with this figure dropping to 7% afterwards.

The participants appear to have been liberated by the course to adopt a more fallibilistic stance. This is borne out by many of the comments made in the post-course survey:

I experienced many light bulb moments.

... the course has had a significant effect on my day to day teaching.

I now teach differently because this course has helped me identify what really matters in maths learning.

The impact on my own class has been enormous.

I have become much more flexible and creative in my teaching.

While there were notable moves from the fundamentalist standpoint, there were no such moves in the other direction. In other words, fallibilist beliefs held were not challenged in any way by participation in the course and indeed the opposite can be assumed as agreement with all statements increased or stayed the same. The greatest shifts noted were in response to the statements 'Maths is about my own creative reasoning' and 'I find maths enjoyable and fulfilling'.

5.3. Interviews

A qualitative approach was taken to the analysis of the interviews, which resulted in the identification of two particularly strong emergent themes, which we describe as 'being allowed' and 'labelling'. The interviews were structured around a number of key issues, which are outlined in the following sections together with a selection of quotations from teachers, which give a flavour of the overall responses.

5.3.1. Pre-course interviews. Four teachers were interviewed prior to taking the course of study, one from each local education authority (Dundee, Fife, Angus and Perth and Kinross). All were female and each one had 10 years or more classroom experience.

The approach taken was to use semi-structured interviews on the themes of:

- confidence and competence in relation to teaching mathematics
- attitudes and beliefs in relation to mathematics as a subject
- and expectation of the impact on their practice and on pupil learning arising from the course of study

In addition to the start of each interview, each teacher was asked to talk about her own personal early educational experiences of mathematics. The interviews produced rich data and similar themes emerged from each one.

5.3.1.1. Early experiences and critical incidents. When asked about their own early experience of mathematics, each teacher gave an account of at least one negative experience which had remained vivid in her memory. Furthermore, the nuances of mathematical fundamentalism were inherent in the discourse used by the teachers when discussing their early educational experiences. For example, Naomi described her own experience as being about 'learning to pass exams', while Alice spoke of not understanding the relevance of mathematics.

Moreover, all four teachers recall having been positioned or labelled in terms of their ability in mathematics. Each teacher described either a symbolic or physical positioning which took place and talked about the

consequence of such positioning. In fact, Angela who started in the ‘top class’ for mathematics at secondary school described how she was ‘pulled out of the classroom’ and ‘very quickly shoved into another one’ where she was not expected ‘to do very much’. Indeed, she described herself as ‘definitely one of these people who have labelled themselves as not a maths person’.

I was and still am I suppose very definitely one of those people who had been traumatised by maths from secondary school ... but actually it sounds so very simplistic and I don’t want to give it a simplistic answer but I think just from some of the examples I have been reading in the books and some of the things I have been reading on the module materials. I have allowed myself to be labelled, and I have labelled myself and I have carried that label on and really that change has to come from me and I am quite intrigued now about getting involved in this module and shaking that off and coming to the end of this ... and being someone who does maths. ... I am quite excited about maybe being someone in my own school who has the competences that will be helpful to other teachers as well, and that is something a million miles away from the person doing higher maths and not understanding a word of it. (Angela)

This particular example illustrates both a negative school experience and it also articulates the strong desire expressed by all four teachers of wanting to ‘make a difference’ to the mathematical experience of their pupils.

5.3.1.2. Confidence and competence. Each teacher talked with enthusiasm about mathematics. Most notably, they each talked about changing and developing as a teacher over time and at the same time developing their confidence in mathematics. This slowly developing confidence in teaching mathematics was gained through classroom experience and was specific to the age range taught—or, as Angela described it—her ‘comfort zone’. All teachers referred in some way to being at a stage when mathematics was ‘becoming clearer’ and while all four teachers were clearly gaining confidence in their own mathematical ability the journey up to this point had taken time (over a period 10 years for all) and was clearly linked to gaining experience as a teacher in general.

5.3.1.3. Expectations of the course. When asked about their expectations of the course, all four teachers interviewed hoped to gain new ideas and develop understanding of new methods. They all expressed a desire to have the confidence to try out new methods and not rely on textbooks. Indeed, one teacher talked about her struggle to ‘step away from the workbook’ and the discourses of ‘control’.

While there was a certain amount of trepidation amongst the teachers regarding the course, they were all highly motivated by the ‘challenge’ and saw both the course and Curriculum for Excellence as ‘opening the door’ to try out new and creative approaches to teaching mathematics which could be shared with the children. In particular, there was an expectation that by bringing theory, research and practice together in the classroom their mathematical activities would gain ‘credibility’ and tacit

knowledge would become explicit and formalized. In addition, it was clear that all four teachers had been supported and encouraged locally and all expressed a desire to 'make a difference' in terms of helping children to understand and enjoy mathematics.

5.3.1.4. Being allowed. A related theme to emerge was that of 'being allowed' which is captured by the following quote from Angela who reflects on her sense of liberation in being allowed to write things down and draw a picture and in turn to allow her pupils to do likewise:

... you know one size does not fit all not everyone is going to understand the chalk and talk. For some people that works very well for them and others are very visual and I am a very visual learner and so my first instinct would be to draw picture, write everything down and I suppose why I am talking about that is I still notice that some children don't think that is allowed that they some how are not allowed to make jottings or recordings it is cheating or something. That they are not allowed to show their working that their answer has to come from here and if it doesn't come from here then they haven't done it properly. (Angela)

This aspect also emerged from the interview with Alice who commented that:

Mathematics at the moment it is asking about things, finding kind of solutions to things, finding out ways to say measure something or explain something, it is not text books, there is a lot of collaboration in it. There is a lot of interactive stuff there has been lots of being allowed to jot down little notes, draw pictures. (Alice)

Naomi also referred to this aspect and to the impact of Curriculum for Excellence in opening up possibilities:

The start for Curriculum for Excellence has definitely opened the door, it is going to allow us to do more things like the Budget Brain thing I mentioned and that is the road I would like to go down. (Naomi)

This was a recurring theme that is returned to and discussed further in the final section.

5.3.2. Post-course interviews. Towards the end of the course, three of the original group of teachers were interviewed with a fourth teacher who volunteered to replace a teacher from the original group who was unable to attend the post-course interviews. In line with the pre-course interviews, each teacher was from a different authority.

5.3.2.1. Building confidence and being allowed. All teachers spoke with enthusiasm about the difference that the course had made to them particularly in terms of gaining confidence in their teaching of mathematics in the classroom. The difference was explained in various ways, for example, emphasis was placed on 'taking time', 'using a variety of methods', 'relating maths to real life' and once again to 'being allowed'. Alice described

how she now had built her confidence by ‘being allowed’ to ‘take time’ over her teaching of mathematics.

I think being allowed to actually take time over a piece of mathematics. (...) it’s not just a case of getting through 15 pages of maths. I’m quite happy now to get through a couple of written pages, I look at the concepts that you are being asked to teach and I tackle them in different ways, I choose a variety of teaching methods and I find the children respond much better to this as well. (Alice)

Naomi explained how the collaborative element of the course had been beneficial in supporting her work with other teachers in her school:

... for my professional development it’s been fantastic. Just to be able to speak to colleagues and have the confidence to say I have done this in my classroom, I know this works and I’m Acting PT at the moment in my school so within my department I have been able to say to staff, you know, I’m happy for you to try something different, ... don’t feel you have to stick to these structures. (Naomi)

Indeed, all four teachers interviewed talked about having the confidence to try new approaches and different ways of teaching mathematics. Naomi talked about teaching through questioning:

... for me, the main difference in my teaching is now I do more asking than telling, ... (Naomi)

Naomi also described her open-ended questioning and her ‘very, very active classroom’.

Sally explained that how, for her, the course had been an affirming process that was now helping her to support mathematical development in her school:

I think that doing this has confirmed to me that I have a pretty good understanding of maths and how to teach it and so, now that my Depute is actually doing the course as well, it’s nice because we want to create a whole school approach. We want to actually take this back to our Head (...) and actually say, right this is what we’ve done, it’s been really useful and this is why. So, we see a future in what was started here, I see a future for it actually going on. (Sally)

5.3.2.2. Benefits to the pupils by ‘being allowed’. All teachers interviewed talked with passion about the positive impact their teaching had had on the children in their classroom. Indeed, there was a sense of revitalization. Chloe talked about having high expectations and creating a relaxed stress-free environment. Alice talked about teaching that ‘has allowed the kids to find out what maths is about’.

‘... they’re actually listening to what I say!’ they’re using this language with you, which is brilliant. I think the main thing is the children’s confidence; they don’t feel that they’re not being allowed to do something. They don’t feel that there’s ... the barrier there. I hope that the children

in my class now know that if they put in the effort and they participate they can go as far as they want. (Alice)

Teresa also spoke of the benefits to the children as a result of them 'being allowed' to find out:

I was kind of challenging myself as I was talking to the kids as I was going round to say I'm not going to ask a question that I can see what the answer is or I wanted to, sort of, stretch them a little bit more and it was great because even the group who had initially, they had thought they knew what the question was, when they actually started working it out and they took all the information that they had, you could see that while the rest of the groups were all really, really busy, you could tell this group was kind of sitting thinking going 'this isn't quite right' so, again, just a little bit more questioning and getting them to go back and think about the different parts of the question and the information that they've got, eventually one wee boy in the group said 'oh, oh I know what it is, we've been doing this and we should have been doing that' and it was, you know, the light bulb moment, the rest of the kids were like 'oh, right, right, right, got it' so I think that I've been trying to teach in a way that has been allowing the kids to find out what the maths is, rather ... you know ... (Teresa)

All teachers interviewed talked about the children gaining confidence and enjoying mathematics. The course was clearly seen by the teachers to be beneficial to the development of mathematics teaching in the classroom. An additional dimension that Alice mentioned was the positive link between the course and the implementation of Curriculum for Excellence.

... with regards to my maths teaching, that has certainly changed and again, ... I would say it was a combination of attending the course it's making me think more about maths and also the introduction of Curriculum for Excellence, (Alice)

6. Conclusions

Findings from this study highlight the way the course had a transformational and emancipatory impact on these teachers. With regard to the change in teachers' perceptions concerning their levels of confidence and competence in relation to teaching mathematics, whilst 65 and 73% of participants already reported that they were good at and confident about mathematics before the start of the course, these figures rose to 93% in both categories by the end of the course. In relation to change in attitudes and beliefs, there was a distinctive move from the fundamentalist viewpoints held by many of the participants at the start of the course, in particular, those relating to the absolutist nature of the subject regarding rules, right and wrong answers and testing.

The interview responses with regard to the teachers' expectations of the impact on pupil learning arising from this course of study were typical of the group as a whole. These teachers hoped to gain new ideas and

develop understanding of new methods. They expressed a desire to have the confidence to try out new methods and not rely on textbooks. While there was a certain amount of trepidation amongst the teachers regarding the course, they were highly motivated by the challenge and saw both the course and Curriculum for Excellence as an opportunity to try out new and creative approaches to teaching mathematics. There was general desire to 'make a difference' in terms of helping children to understand and enjoy mathematics.

The two key texts that were selected as key readings provided a challenge to widely held views about the nature of mathematics and also provided additional support for the development of active and participatory teaching methods. In her book, Boaler (2009) refers to a narrow subject which involves copying methods that teachers demonstrate and reproducing them accurately over and over again and which she argues is not mathematics but rather which is 'a strange mutated version of the subject' that is taught in schools. This process of 'mutation' reflects the process of didactic transposition, which changes the mathematical knowledge profoundly and which leads to the *epistemic quality* of the subject becoming degraded as it is transposed into school mathematics. We describe this mutated or degraded version of mathematics as *mathematical fundamentalism* and as being of *low epistemic quality*. It is characterized by an approach that presents the subject as infallible, authoritarian, dogmatic, absolutist, irrefutable and certain and which involves rule following of strict procedures and right or wrong answers. This is not a criticism of mathematical formalism, or of the deductivist approach, but rather of the result of the 'mutation' of the subject into an all too common form of degraded school mathematics of low epistemic quality through the process of didactic transposition.

The findings also highlight ways in which the 'framing' (Bernstein, 2000) of particular aspects of the traditional curriculum had an oppressive impact on learners in the ways that suppressed creativity and limited the exercise of learner autonomy through what is perceived to 'be allowed' or 'not allowed' by both teachers and pupils. It is clear that the weaker framing of Curriculum for Excellence shifted the locus of control over the selection, sequencing and pacing of what counts as legitimate knowledge towards these teachers. However, the teachers' own experience as learners of mathematics highlights the impact of the strong framing over the criteria for evaluating or assessing of the formal assessment system, especially at secondary school level. Accordingly, an ongoing challenge for continuing reform will be the alignment criteria for evaluating or assessing of the formal assessment system with the aims and purposes of the formal curriculum.

The responses from this group of teachers provide strong affirmation of the arguments presented by Boaler (2009) with many examples for profoundly negative experiences as early learners of mathematics. Each teacher who was interviewed gave an account of at least one negative experience which had remained vivid in her memory, when asked about her own early experience of mathematics. Furthermore, the nuances of mathematical fundamentalism were inherent in the discourse used by the

teachers when discussing their early educational experiences e.g. about learning to pass exams and of not understanding the relevance of mathematics. Similar experiences were shared in the discussion forum on the nature of mathematics and also in the face-to-face meetings. All four teachers recall having been positioned or labelled in terms of their ability in mathematics. Each teacher described either a symbolic or physical positioning which took place and talked about the consequence of such positioning. In analysing this through the lens of Bourdieu's concept of symbolic violence, we can see how this operated both as a 'soft' form of violence by the teacher as a holder of capital as a means of limiting and controlling the agency of individuals with less capital and also as a not so soft form of violence in the case of Angela who described how she was 'pulled out of the classroom' and 'very quickly shoved into another one'. Also, one can see through these accounts how symbolic violence becomes the violence, which is exercised upon a social agent with her complicity, in the way in which a number of these teachers as early learners of mathematics labelled themselves as being 'not able to do mathematics'. Furthermore, viewing these accounts through the lens of 'symbolic violence' illuminates the 'Elephant in the Room' very brightly and affirms the proposition by Boaler that success in mathematics is not a sign of general intelligence and also supports her in her refutation of the proposition that some people can do and some can't do mathematics.

Acknowledgement

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